

Semi-analytic Methods for Modelling Photonic Woodpiles

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Declaration of Originality

I certify that the work in this thesis has not previously been submitted for a degree nor has it been submitted as part of requirements for a degree. I also certify that the thesis has been written by me. Any help that I have received in my research work and the preparation of the thesis itself has been acknowledged. In addition, I certify that all information sources and literature used are indicated in the thesis.

Signature of Author

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Abstract

New semi-analytical methods are presented for modelling the electromagnetic fields of three-dimensional photonic crystals that are composed of orthogonal layers of cylindrical rods. Firstly, the *multipole method* is extended so that cylindrical defects, which act as optical waveguides, can be introduced into such ‘woodpile’ structures. These waveguides are important because they offer greater control over the mode dispersion and optical losses than conventional waveguides do. The multipole method forms the basis of all of the techniques presented in this thesis, and is employed here because it is considerably faster than the pre-existing methods for modelling woodpile waveguides.

Two approaches for modelling linear defects are presented. The first approach uses a grating super-cell to approximate a localised defect, and results are presented for both a coupled resonator optical waveguide and for a linear waveguide, where each waveguide is embedded in a finite woodpile cladding. The existence of waveguiding modes is inferred from the transmission spectra, and is verified by numerically reconstructing the fields. Furthermore, low loss waveguiding is observed for the linear waveguide.

To complement the super-cell approach, we have generalised the two-dimensional *fictitious source superposition* method, whereby the defect modes of a woodpile are computed directly. The principal advantage of this approach is that it is particularly efficient, making this approach well-suited to the task of tuning the dispersion relationships of the defect states; however, the performance gains are achieved by forgoing the ability to deal with finite structures. The dependence of the dispersion on the refractive index and size of the defect is investigated, and it is shown that tuning these parameters is an effective method for optimising the waveguide for operation in the slow-light regime.

Lastly, a comprehensive analysis of the surface modes of photonic woodpiles is performed. Specifically, the surface modes of both finite and semi-infinite woodpiles are characterised using transfer matrix and plane wave matrix formulations. In the case of finite structures, a general mathematical description of the modes that propagate simultaneously along the top and bottom surfaces is given. It is shown that when the number of layers is even, such ‘double-interface’ modes only exist for specific directions of the Brillouin zone. However, when the number of layers is odd, every surface mode is a double-interface mode and, in this case, the direction of propagation plays an important role in determining the coupling strength between the two surfaces: for certain directions, the coupling is negligible even when the number of layers is small. The dispersion curves

of two different double-interface modes can anticross or be interwoven, depending on the symmetry of the modes. A Fabry-Pérot cavity comprising two woodpile barrier regions is also considered. In particular, the conditions required in order for coupled surface modes to exist in these ‘compound woodpile’ structures are described.